Teaching Mathematics Vocabulary Using Hands-On Activities

From an MSP Grant Summer Institute

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Strand III-Professional Development

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A Merriam-Webster dictionary gives the following definition:

> "VOCABULARY--1**:** a list or collection of words or of words and phrases usually alphabetically arranged and explained or defined: 2 a**:** a sum or stock of words employed by a language, group, individual, or work or in a field of knowledge b**:** a list or collection of terms or codes available for use."

Foreign languages courses have always recognized the importance of students learning the vocabulary for the language. It is impossible to master the language without first having a thorough knowledge of the vocabulary. According to the above definition, the collection of words must be explained and they must be available for use. Unfortunately, students of mathematics have many times not mastered the new vocabulary in this "language of the sciences". The lack of knowledge of the vocabulary has often prevented students using mathematics as a tool in solving problems in areas of application.

Recently many school systems have become aware of the lack of the mastery of the vocabulary of mathematics both by their students and by teachers in the classroom. As a result, many systems have initiated special workshops and professional development opportunities to encourage the development of a strong mathematics vocabulary. According to Robert Marzano & Debra Pickering (2005) " the best way to teach academic vocabulary from content areas is to observe the following steps:

1- provide a description, explanation, or example of the new term
2- ask students to restate the description, explanation, or example in their own words
3- ask students to construct a picture, symbol, or graphic representing the term or phrase
4- engage students periodically in activities that help them add to their knowledge of the terms in their notebooks
5- periodically ask students to discuss the terms with one another
6- involve students periodically in games that allow them to play with terms."

In keeping with the spirit of item 3 in the guidelines from Marzano and Pickering, the SEE-Math (**S**tudent **E**ngagement in **E**xploring Math) Institutes (supported by a Math/Science Partnership Grant) at Lipscomb University in Nashville, Tennessee have focused on building mathematics vocabulary through the use of hands-on activities. Thirty-four high school teachers and ten middle school teachers participated in the summer '07 institutes. Of these, twenty-seven were in Institute X, *Statistics, Geometry, and Problem Solving*, where they used *Geometer's Sketchpad, Algebra in Motion, and MINITAB,* did hands on activities with paper folding, and focused on problem solving techniques. The other seventeen teachers were in Institute Y, *Pre-Calculus and Calculus,* where they used *MAPLE 11* and *Calculus in Motion,* used hands-on activities to measure

heights, studied the mathematics associated with rockets and trajectories, and created fractals using Newton's Method for finding roots of equations. In both institutes, all activities were correlated with the Tennessee Learning Expectations for the content areas.

Geometry is one area where the use of hands-on activities especially in the form of paper folding is easily done. One of the activities which is popular and which has been presented at several mathematics conference is "Folding a Circle". Usually this activity is never developed to the fullest in these presentations. However, "Folding a Circle" has been expanded for use in the SEE-Math Institute X in such a way that over 80 geometric terms are defined and illustrated in the single hands-on activity. The instructions for the expanded version of this hands-on activity are given below along with a list of vocabulary to discuss at each step.

FOLDING A CIRCLE

USE : During the activity, over eighty geometric terms can be defined and illustrated. With elementary level students, only the basic terms should be discussed; however, there are concepts appropriate for university and graduate level students also.

SOME MATHEMATICS TO DISCUSS:

During the activity, all of the following terms or concepts can be reviewed. .

Rectangle
Quadrilateral
Parallelogram
Parallel lines
90 degree angle
Plane
Measure
Area of a rectangle
Perimeter
Circle
Two dimensional disk
Area of a circle
Circumference of a circle
Irregular polygon
Line segment
Diameter
Semicircle
Center of circle
Radius
Chord

Triangle
Equilateral triangle
Isosceles triangle
Area of a triangle
Pythagorean theorem
Trapezoid
Parallel vs non parallel
Polygon
Isosceles trapezoid
Rhombus
Length
Similar
Congruent triangles
Fractions
Pyramid
Tetrahedron
Platonic solid
Surface area
Pentagon
Regular Polygon
Hexagon
Central angles of polygons
Sum of the measures of the interior angles of a polygon
Exterior of a set
Truncated tetrahedron
Common denominator

Equiangular triangle
Sum of measures of the angles in a triangle = 180 degrees
Base
Vertex
Point
Altitude
Median
Circumcenter
Incenter
Orthocenter
Centroid
Angle bisector
Perpendicular bisector
Scalene triangle
Right triangle
Hypotenuse
Leg of a right triangle
30- 60-90 triangle
Incircle
Polyhedron
Arithmetic of fractions
Icosahedron
Closed set
Bounded set
Compact set
Interior of a set
Quadrants
Secant line
Arc of a circle
Euler Line
Sector of a circle
Midpoint of a line segment
Triangle inscribed in a circle
Inscribed angle
Central angle
Acute angle
Obtuse angle
Diagonal
Lines of symmetry
Circumcircle

MATERIALS NEEDED: One 8.5"x11" sheet of colored paper with as large a circle as possible constructed on one side; scissors; tape.

INSTUCTIONS:

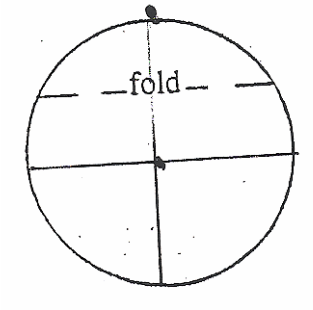
1. Discuss all mathematical terms and concepts which are illustrated by the blank side of the paper. Include rectangle, quadrilateral, parallelogram, line segment, parallel lines, right angle, area of a rectangle, perimeter of a rectangle, diagonal of a rectangle, plane, and polygon.

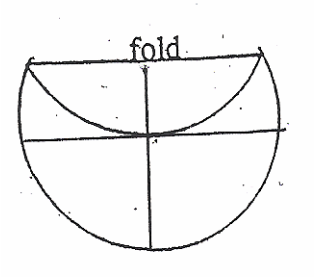
2. Discuss all mathematical terms and concepts which are illustrated by the side of the paper with the circle constructed on it. Include circle, area of a circle, circumference of a circle, interior of the circle, exterior of the circle, two dimensional disk, and (if a student knows some Euclidean topology) closed set, bounded set, and compacat set.

   Cut out the circular disk and discard the scraps. Discuss lines of symmetry for the circular disk.

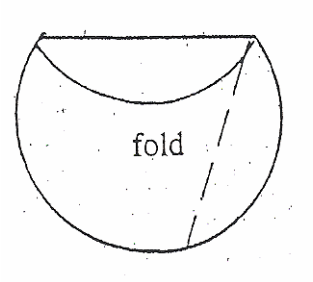
3. Approximate the center of the circular disk with a pencil. Fold the disk in half and unfold. Discuss diameter and semicircle. Refold and fold the semicircle in half and unfold. Discuss line of symmetry, radius, center of the circle, central angle, and quadrant.

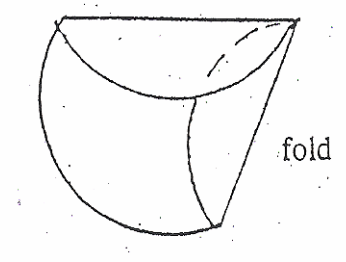
4. Mark a point on the circle at the end of one of the radii formed in step 3. Fold the point to the center, folding the radius on itself, and unfold. Discuss point, chord, and sector of a circle.



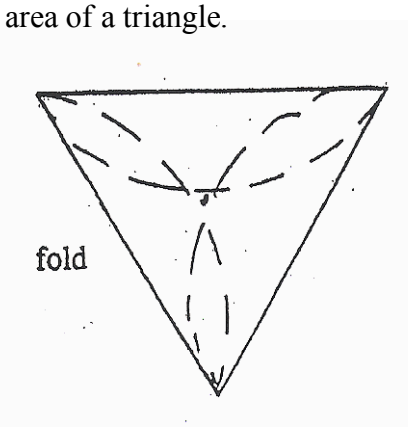


. 5. Fold another point on the circle to the center, using one endpoint of the chord formed in step 4 as an endpoint for a new chord and unfold. Discuss inscribed angle





6. Fold the remaining arc of the circle to the center. Discuss triangle, arc, equilateral triangle, isosceles triangle, scalene triangle, equiangular triangle, sum of the measures of the angles of a triangle, base of a triangle, vertex, acute angle, obtuse angle, triangle inscribed in a circle, and area of a triangle.

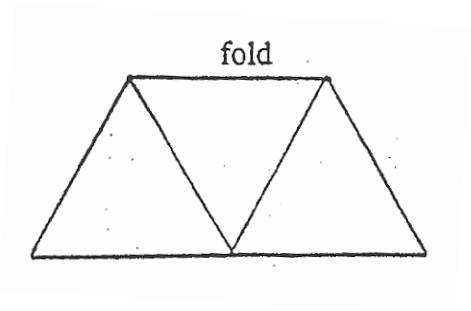


7. Fold the triangle on one of its line of symmetry. Discuss right triangle, 30-60-90 triangle, Pythagorean Theorem, and angle bisector. Unfold and discuss altitude, median, and perpendicular bisector of the base.

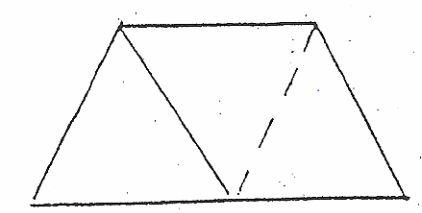
8. Fold the triangle on all three lines of symmetry, unfolding after each fold. Notice where the three lines of symmetry crossed. Discuss centroid, circumcenter, orthocenter, and incenter. Construct the incircle and placing the triangle on a piece of paper, construct the circumcircle. Discuss the lack of an Euler Line in an equilateral triangle.

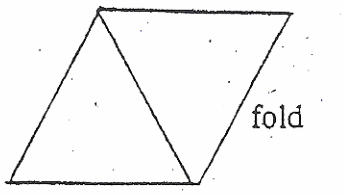
In the remaining steps, assume that the area of the original equilateral triangle is one square unit.)

9. Fold to find the midpoint of one of the sides of the triangle. Fold the opposite vertex to this midpoint. Discuss measure, midpoint of a line segment, trapezoid, isosceles trapezoid, and area of the trapezoid. **(Remember that the area of the original equilateral triangle was assumed to be one square unit.)**

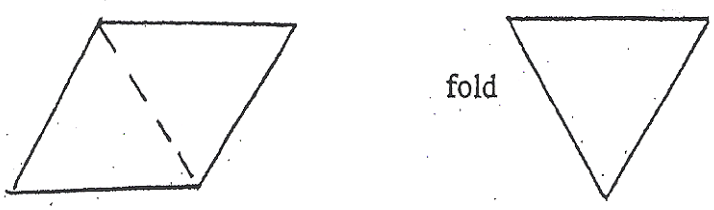


10. Notice that the trapezoid consists of three congruent triangles. Fold one of triangles over the top of the middle triangle. Discuss rhombus, congruent triangles, and area of the rhombus. **(Remember that the area of the original equilateral triangle was assumed to be one square unit.)**

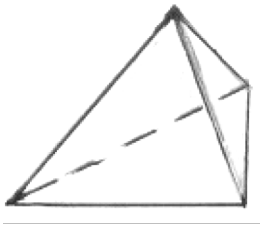




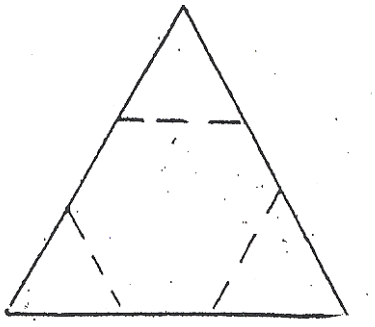
Then fold the rhombus on a line of symmetry and discuss the area of this new equilateral triangle and its relationship to the earlier equilateral triangle (similar).

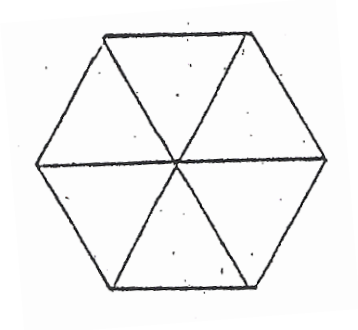


11. Open up the three folded over triangles until their corners meet forming a three dimensional figure. Discuss polyhedron, tetrahedron, Platonic Solid, pyramid, and surface area of the tetrahedron. **(Remember that the area of the original equilateral triangle was assumed to be one square unit.)**



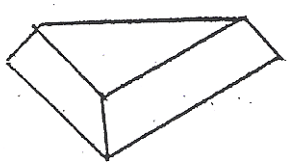
12. Open to the large equilateral triangle made in step 6. Fold each of the vertices to the center of the circle. Discuss irregular pentagon (formed when the second corner was folded to the center), hexagon, regular polygon (hexagon), sum of the measures of the interior angles of a polygon, and area of the irregular pentagon and the regular hexagon . **(Remember that the area of the original equilateral triangle was assumed to be one square unit.)**





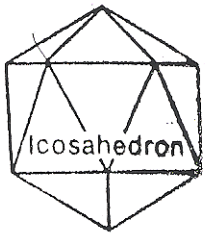
13. Bring the small triangles in the middle of the hexagon formed in step 11 so that they are on top of each other and a truncated tetrahedron is formed.. Discuss truncated tetrahedron and find its surface area. In

finding the surface area, review common denominator and the addition of fractions.



14. Using only the fold lines already determined, create different polygonal figures and determine their areas. Using only the existing fold lines, construct figures with the following areas: 1/4, 1/2, 19/36, 2/3, 3/4, 7/9, 8/9, 7/18, 23/36 square units. Remember that the area of the triangle formed in step 6 is one square unit.

15. Tape the top of the truncated tetrahedron . **NOW REPEAT STEP 1-13 TO FORM NINETEEN MORE OF THESE TRUNCATAED TETRAHEDRONS.** Tape the twenty truncated tetrahedra together to make an icosahedron (the Platonic Solid which has twenty triangular faces). This is best done by taping five of the truncated tetrahedra together to form a "top", five to form a "bottom", and ten to form a "band" for the center. Then tape the top and bottom to the center band. Before taping the top to the center band, notice the regular pentagon formed by the edges on the underside of the top.



NOTE: The icosahedron is the form that the AIDS virus takes as well as many other well known viruses. At this stage, students can be asked to do research on the icosahedron and integrate biology and mathematics. Also, the icosahedron is one of the five Platonic Solids. This could generate a new area of study of these unique three-dimensional figures.

Many other hands-on activities are included in the two-week institutes sponsored under this Math/Science Partnership grant. Institute participants are implementing these activities in their classrooms and some participants have made these comments:

- "I cannot begin to tell you how much I have grown as a teacher because of this workshop. I feel more confident. I feel better equipped to teach concepts without relying so much on the textbook."

- "The most effective way that I learn and teach is hands-on. The geometry activities solidified the pedogological learning and understanding."

- "I will use the problem solving and the geometry activities in my class. I will use what I have learned in statistics to deepen my students' understanding of basic measures of central tendency."

- "I will be sharing the notebook with my department head who teaches geometry and has asked for ideas."

- "I loved the paper folding. I was not excited about teaching geometry this fall, but I am so excited about all the new things I have learned."

All participants agree that students are being engaged in exploring mathematics.

References

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Merriam-Webster Dictionary. Retrieved August 23, 2007, from http://MW1.Merriam-Webster.com/dictionary/vocabulary.